

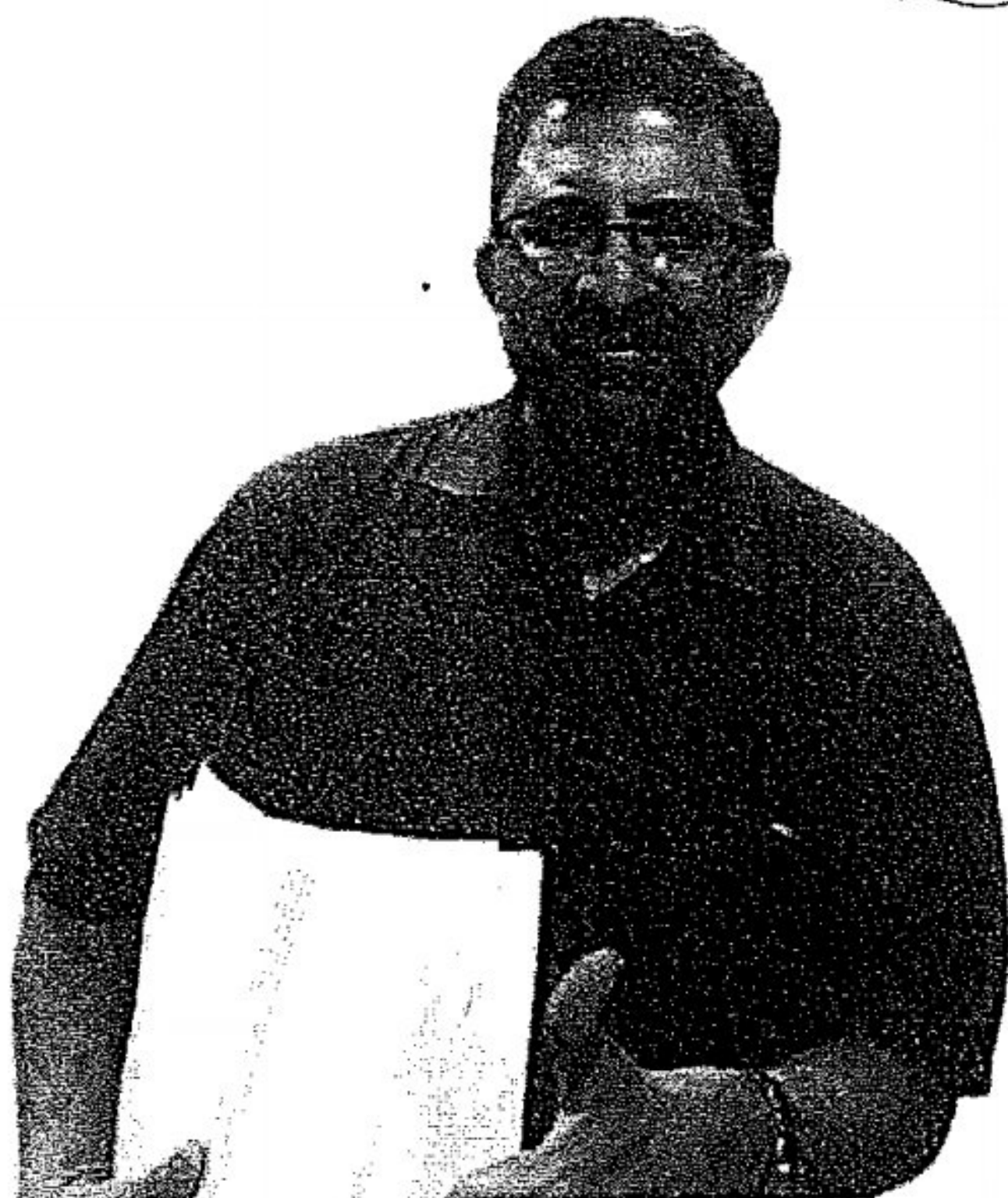
ADDMATHS

DIFFERENTIATION

(PEMBEZAAN)

PAPER 2 (*KERTAS 2*)

SPM



QUESTION 1

TRIAL
NEGERI KEDAH

- (a) Cari persamaan normal bagi lengkung $y = (2x - 3)^4$ pada titik $(1, -3)$.

Find the equation of the normal to the curve $y = (2x - 3)^4$ at the point $(1, -3)$.

[3 markah / marks]

- (b) Suatu lengkung dengan fungsi kecerunan $3x - \frac{3}{x^2}$ mempunyai titik pusingan di $(m, 7)$

A curve with gradient function $3x - \frac{3}{x^2}$ has a turning point at $(m, 7)$.

- (i) Cari nilai m ,

Find the value of m ,

- (ii) Tentukan sama ada titik pusingan ini adalah titik maksimum atau titik minimum.

Determine whether the turning point is a maximum or minimum point.

[4 markah / marks]



QUESTION 2

(a) Diberi $f(x) = \frac{3x}{2 - \sqrt{x+4}}$,

Given that $f(x) = \frac{3x}{2 - \sqrt{x+4}}$,

(i) cari $f(0)$,

find $f(0)$,

(ii) lengkapkan Jadual 3(i) dan Jadual 3(ii) yang diberi dalam ruang jawapan.
Berikan jawapan anda betul kepada empat tempat perpuluhan.

complete the Table 3(i) and Table 3(ii) given in the answer space.

Give your answers correct to four decimal places.

x	-0.1	-0.01	-0.001	-0.0001	0
$f(x)$					-

Jadual 3(i)

Table 3(i)

x	0.1	0.01	0.001	0.0001	0
$f(x)$					-

Jadual 3(ii)

Table 3(ii)

(iii) daripada jawapan dalam Jadual 3(i) dan Jadual 3(ii), nyatakan nilai bagi

had $\frac{3x}{2 - \sqrt{x+4}}$ $\lim_{x \rightarrow 0}$

from the answer in the Table 3(i) and Table 3(ii), state the value of

$\lim_{x \rightarrow 0} \frac{3x}{2 - \sqrt{x+4}}$



QUESTION 2

(b) Diberi $y = \frac{x^2 - 4}{x}$, ungkapkan $x^3 \left(\frac{d^2 y}{dx^2} \right) + x^2 \left(\frac{dy}{dx} \right)$ dalam sebutan x .

Kemudian, carikan julat bagi nilai x di mana $x^3 \left(\frac{d^2 y}{dx^2} \right) + x^2 \left(\frac{dy}{dx} \right) < 0$.

Given that $y = \frac{x^2 - 4}{x}$, express $x^3 \left(\frac{d^2 y}{dx^2} \right) + x^2 \left(\frac{dy}{dx} \right)$ in terms of x .

Hence, find the range of values of x such that $x^3 \left(\frac{d^2 y}{dx^2} \right) + x^2 \left(\frac{dy}{dx} \right) < 0$.



QUESTION 3

TRIAL
SELANGOR (SET 1)

(a) Diberi $f(x) = \frac{2x}{3 - \sqrt{x+9}}$,

Given that $f(x) = \frac{2x}{3 - \sqrt{x+9}}$,

(i) cari $f(0)$.
find $f(0)$.

(ii) lengkapkan Jadual 2(i) dan Jadual 2(ii) yang diberi dalam ruang jawapan.
Berikan jawapan anda betul kepada empat tempat perpuluhan.
complete the Table 2(i) and Table 2(ii) given in the answer space.
Give your answers correct to four decimal places.

x	-0.1	-0.01	-0.001	-0.0001	0
$f(x)$					-

Jadual 2(i)
Table 2(i)

x	0.1	0.01	0.001	0.0001	0
$f(x)$					-

Jadual 2(ii)
Table 2(ii)

(iii) daripada jawapan dalam Jadual 2(i) dan Jadual 2(ii), nyatakan nilai bagi

had $\frac{2x}{3 - \sqrt{x+9}}$ $x \rightarrow 0$.

from the answer in the Table 2(i) and Table 2(ii), state the value of

$\lim_{x \rightarrow 0} \frac{2x}{3 - \sqrt{x+9}}$.



QUESTION 3

TRIAL
SELANGOR (SET 1)

(b) Diberi $y = x(x^2 - 4)$, ungkapkan $x \frac{d^2y}{dx^2} + \frac{dy}{dx}$ dalam sebutan x .

Kemudian, carikan julat bagi nilai x di mana $x \frac{d^2y}{dx^2} + \frac{dy}{dx} > 0$.

Given that $y = x(x^2 - 4)$, express $x \frac{d^2y}{dx^2} + \frac{dy}{dx}$ in terms of x .

Hence, find the range of values of x such that $x \frac{d^2y}{dx^2} + \frac{dy}{dx} > 0$.



QUESTION 4

TRIAL
NEGERI KELANTAN

- (a) Dua nombor h dan k adalah berhubung secara $h+k=6$. Cari nilai h dan nilai k supaya fungsi $Z=2h^2+3k^2$ mempunyai satu titik pegun. Seterusnya, tentukan sama ada titik pegun tersebut ialah titik maksimum atau minimum.

Two numbers h and k are connected by the relation $h+k=6$. Find the value of h and k which give a turning point of the function $Z=2h^2+3k^2$. Hence, determine whether that stationary point is maximum or minimum point.

[5 markah]

[5 marks]

- (b) Cari koordinat titik pada lengkung $y=(3x-1)^2$ dengan keadaan kecerunan normal kepada lengkung pada titik itu ialah $-\frac{1}{6}$.

Find the coordinates of the points on the curve $y=(3x-1)^2$ such that the normal gradient to the curve at that point is $-\frac{1}{6}$.

[3 markah]

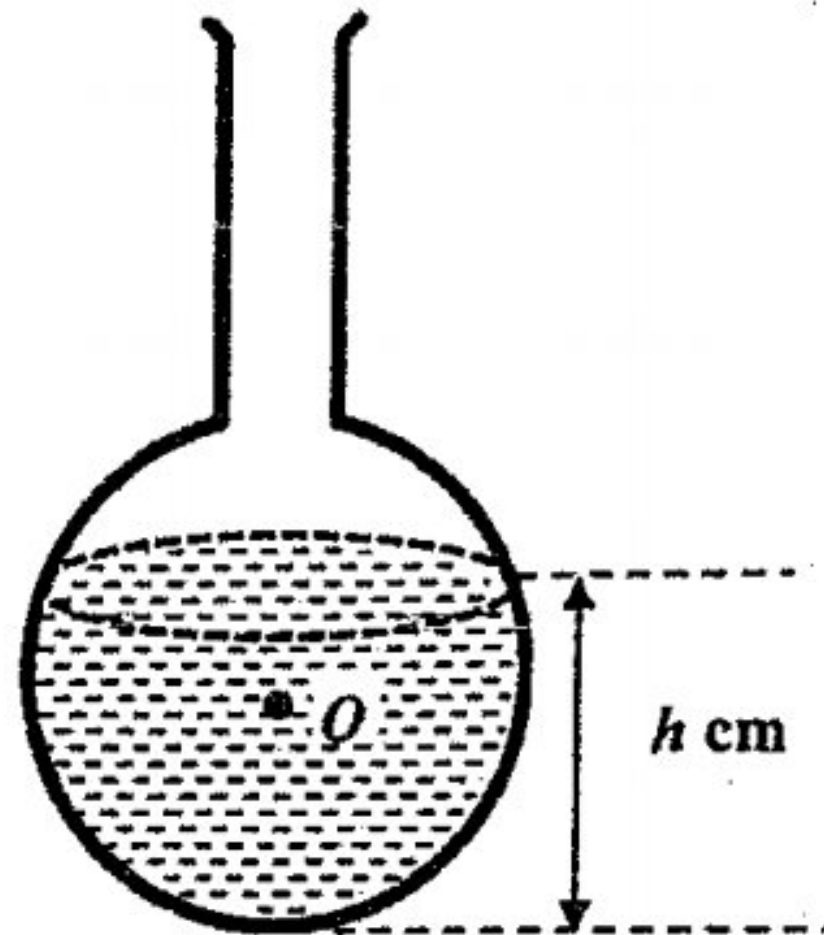
[3 marks]



QUESTION 5

Rajah 6 menunjukkan sebuah kelalang dasar bulat yang berpusat di O berjejari 10 cm. Air dimasukkan ke dalam kelalang itu dengan kedalaman air dari dasarnya, h cm, meningkat dengan kadar 0.5 cm s^{-1} .

Diagram 6 shows a round bottom flask with centre O that has a radius of 10 cm. Water is poured into the flask such that the depth of water from its base, h cm, increases at a rate of 0.5 cm s^{-1} .



Rajah 6
Diagram 6

- (a) Tunjukkan luas permukaan air, $A \text{ cm}^2$, diberikan oleh $A = \pi(20h - h^2)$.
Show that the surface area of water, $A \text{ cm}^2$, is given by $A = \pi(20h - h^2)$.
[3 markah]
[3 marks]
- (b) Cari kadar perubahan luas permukaan air, $\text{cm}^2 \text{ s}^{-1}$, pada ketika kedalaman air ialah 12 cm, dalam sebutan π .
Find the rate of change of the water surface area, $\text{cm}^2 \text{ s}^{-1}$, when the depth is 12 cm, in terms of π .
[4 markah]
[4 marks]
- (c) Apabila $h=12$, terdapat perubahan kecil dalam h sebanyak $p\%$. Dengan menggunakan kaedah pembezaan, cari perubahan kecil bagi luas permukaan air, $A \text{ cm}^2$, dalam sebutan p dan π .
When $h=12$, there is a small change in h by $p\%$. By using the method of differentiation, find the small change of the water surface area, $A \text{ cm}^2$, in terms of p and π .
[3 markah]
[3 marks]



QUESTION 6

TRIAL
SBP (ASRAMA)

- (a) Diberi bahawa persamaan suatu lengkung ialah $y = x^3 - 12x$. Satu garis lurus menyentuh lengkung itu pada titik $K(3, -9)$. Cari persamaan garis lurus itu.
It is given that the equation of a curve is $y = x^3 - 12x$. A straight line touches the curve at point $K(3, -9)$. Find the equation of the straight line.

[3 markah]

[3 marks]

- (b) Diberi bahawa $x + 2y = 4$. Cari nilai minimum bagi P jika $P = x^2 + xy - y^2$.
It is given that $x + 2y = 4$. Find the minimum value of P if $P = x^2 + xy - y^2$.

[4 markah]

[4 marks]



QUESTION 7

TRIAL
NEGERI KELANTAN

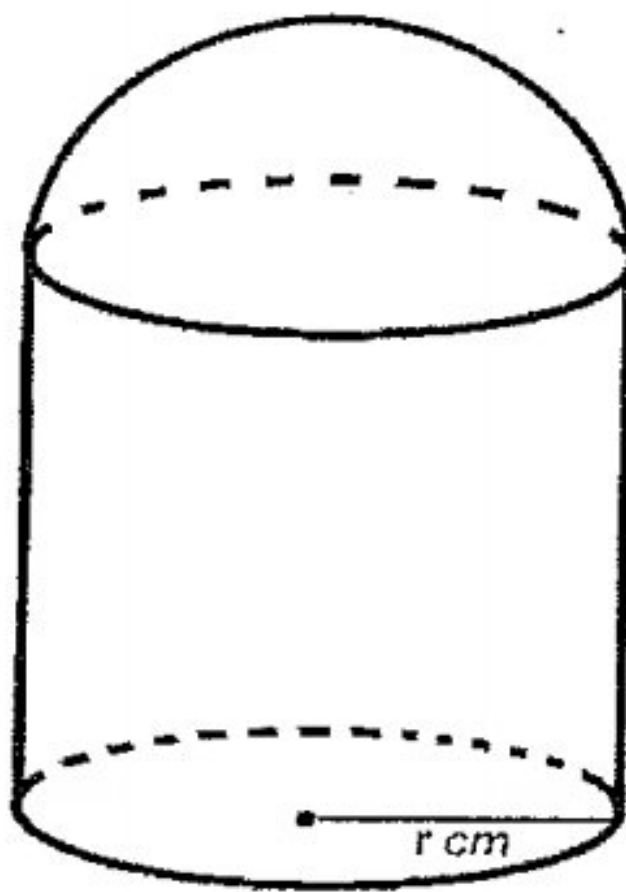
- (a) Cari persamaan tangen kepada lengkung $y = 3x^3 - \frac{1}{x}$ pada titik $(-1, 4)$.

[3 markah]

Find the equation of tangent to the curve $y = 3x^3 - \frac{1}{x}$ at the point $(-1, 4)$. [3 marks]

- (b) Rajah 5 menunjukkan sebuah bongkah yang terdiri daripada satu silinder, dengan jejari r cm dan satu hemisfera di atasnya.

Diagram 5 shows a solid that consists of a cylinder, of the radius r cm, surmounted by a hemisphere.



Rajah 5

Diagram 5

Diberi isipadu silinder itu ialah $24\pi \text{ cm}^3$

Given that the volume of the cylinder is $24\pi \text{ cm}^3$.

[Luas permukaan sfera / Surface area of sphere = $4\pi r^2$]

- (i) Tunjukkan bahawa jumlah luas permukaan bongkah itu, $A \text{ cm}^2$,

diberi oleh $A = 3\pi r^2 + \frac{48\pi}{r}$

Show that the total surface area of the solid, $A \text{ cm}^2$, is given by $A = 3\pi r^2 + \frac{48\pi}{r}$

- (ii) Cari nilai minimum bagi jumlah luas permukaan bongkah itu.

Find the minimum value of the total surface area of the solid.

- (iii) Cari perubahan kecil bagi A , dalam sebutan π , jika r berubah daripada 4 kepada 4.01.

Find the small changes of A , in terms of π , if r changes from 4 to 4.01.

[7 markah]

[7 marks]



QUESTION 8

TRIAL
NEGERI KELANTAN

- (a) Diberi $y = x(3 - x)$, cari nilai x yang memuaskan persamaan $y \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + 12 = 0$

[4 markah]

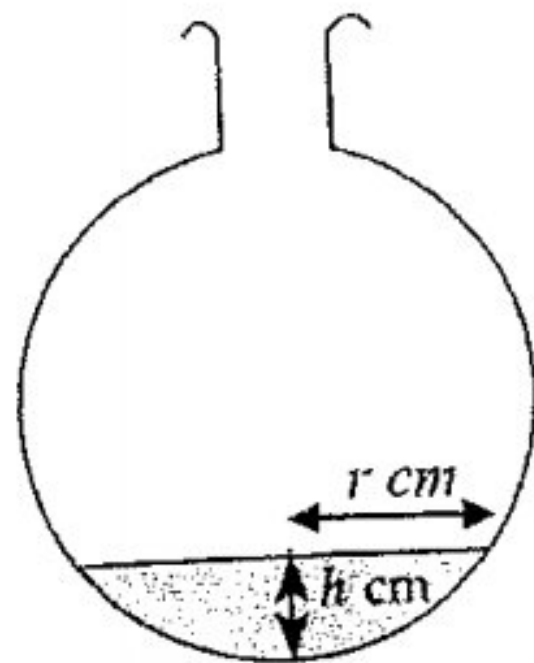
Given that $y = x(3 - x)$, find the value of x that satisfy the equation

$$y \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + 12 = 0 .$$

[4 marks]

- (b) Rajah 3 menunjukkan sebuah kelalang dasar bulat dengan jejari 8 cm. Tinggi paras air di dalam kelalang itu ialah h cm.

Diagram 3 shows a round-bottom flask with a radius of 8 cm. The height of the water level in the flask is h cm.



Rajah 3

Diagram 3

- (i) Tunjukkan bahawa luas permukaan membulat air, $A \text{ cm}^2$, diberi oleh $A = \pi(16h - h^2)$.

Show that the area of the circular water surface, $A \text{ cm}^2$, is given $A = \pi(16h - h^2)$

- (ii) Kelalang itu didapati bocor dan paras air menyusut pada kadar 0.5 cms^{-1} . Hitung kadar perubahan luas permukaan membulat air itu apabila $h = 5$. Beri jawapan anda dalam sebutan π .

The flask was found leaking and the water level was decreasing at a rate of 0.5 cms^{-1} . Calculate the rate of change of the area of circular water surface when $h = 5$. Give your answer in terms of π .

[4 markah]

[4 marks]



QUESTION 9

SPM 2021

- (a) Lengkung $y = \frac{p}{x} + qx$, dengan keadaan p dan q ialah pemalar, mempunyai titik maksimum $(-3, -24)$.

Cari nilai p dan nilai q .

[5 markah]

The curve $y = \frac{p}{x} + qx$, such that p and q are constants, has a maximum point $(-3, -24)$.

Find the value of p and of q .

[5 marks]

- (b) Setitik dakwat yang dijatuhkan ke atas sekeping kertas mengembang dalam bentuk bulatan.

A drop of ink falls on a piece of paper expands to a circular shape.

- (i) Jika jejari bulatan dakwat itu bertambah dengan kadar malar sebanyak 18 mm dalam masa 6 saat, cari kadar perubahan luas, dalam $\text{mm}^2 \text{s}^{-1}$, bulatan itu dalam sebutan π apabila jejarinya ialah 5 mm.

If the radius of the ink circle increases at a constant rate of 18 mm in 6 seconds, find the rate of change of area, in $\text{mm}^2 \text{s}^{-1}$, of the circle in terms of π when its radius is 5 mm.

- (ii) Menggunakan konsep pembezaan, cari nilai hampir bagi luas, dalam mm^2 , kawasan dakwat itu dalam sebutan π apabila jejarinya ialah 5.02 mm.

Using the concept of differentiation, find the approximate value of the area, in mm^2 , of the ink in terms of π when its radius is 5.02 mm.

[5 markah]

[5 marks]

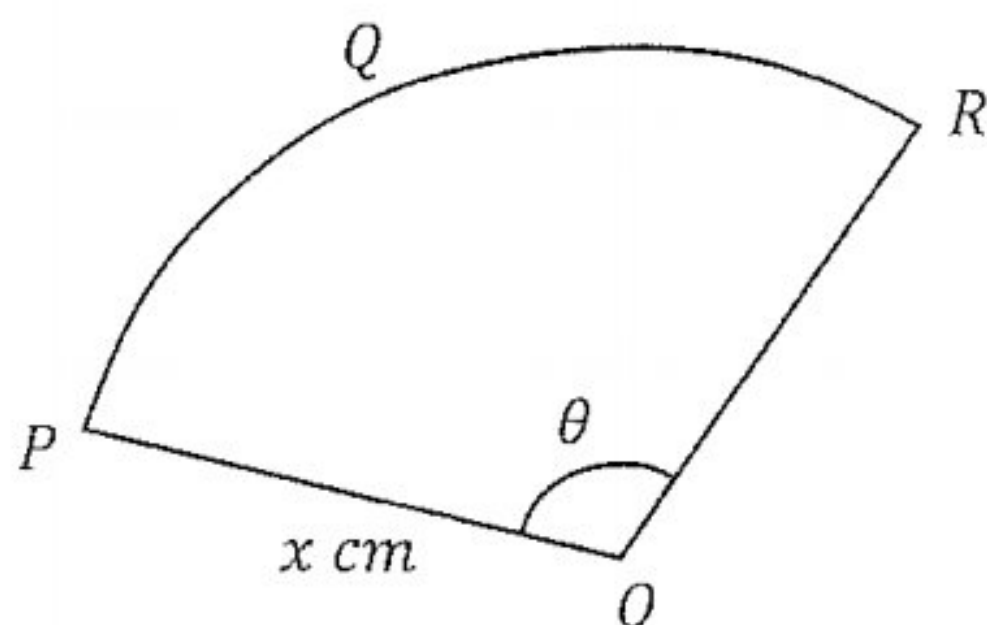


QUESTION 10

TRIAL
NEGERI PAHANG

Rajah 2 menunjukkan seutas dawai yang dibengkokkan menjadi perimeter sebuah sektor bulatan $OPQR$ berpusat O dan berjajari x cm dengan keadaan sudut POR adalah θ radian. Panjang dawai adalah 100 cm manakala x dan θ akan berubah.

Diagram 2 shows a piece of wire bent to form the perimeter $OABC$ of a sector of a circle, centre O , radius x cm, where angle AOC is θ radian. The wire is of length 100 cm while x and θ may vary.



Rajah 2 / Diagram 2

- (a) Tunjukkan bahawa luas sektor, A cm^2 boleh diungkapkan sebagai

$$A = 50x - x^2.$$

Show that the area of the sector, A cm^2 can be expressed as $A = 50x - x^2$.

- (b) Seterusnya, cari

Hence, find

- (i) nilai x supaya luas yang dilingkungi oleh dawai adalah maksimum.
the value of x for which the area enclosed by the wire is a maximum.
- (ii) nilai sepadan bagi θ dalam radian.
the corresponding value of θ in radian.

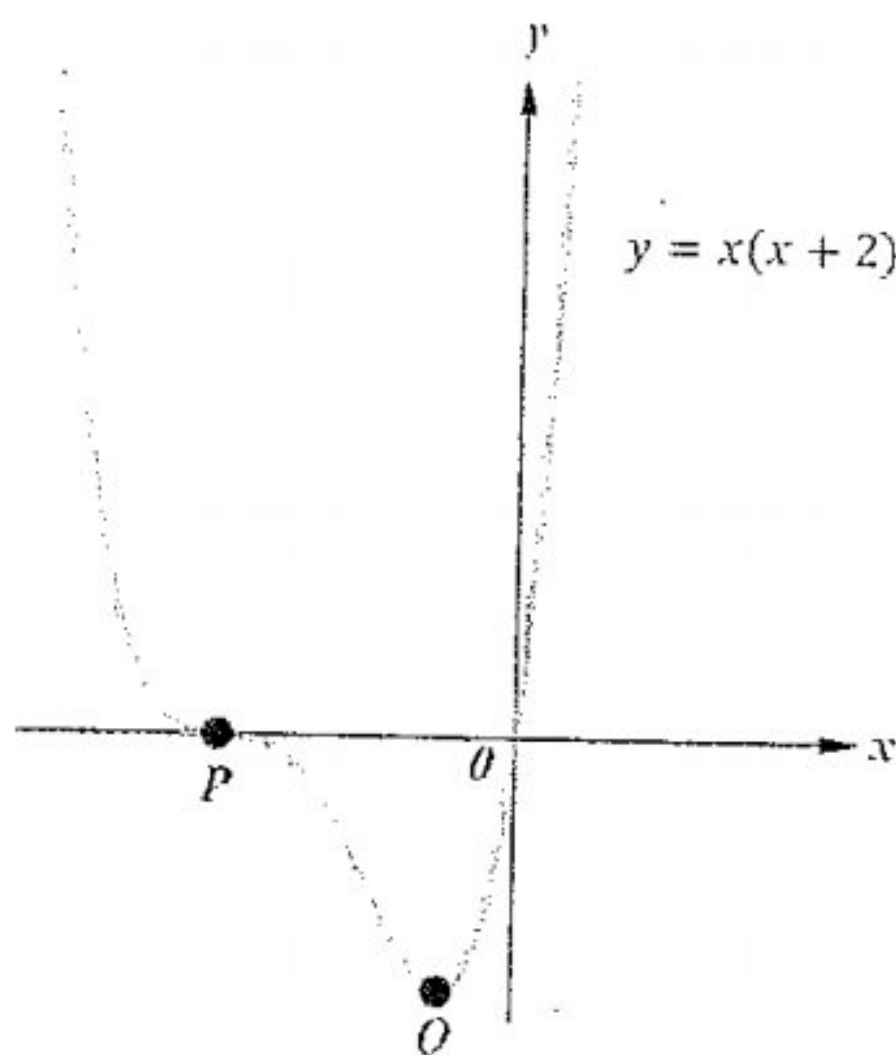
[8 markah / marks]



QUESTION 11

TRIAL
NEGERI JOHOR

Rajah 7 di bawah menunjukkan sebahagian daripada lengkung $y = x(x + 2)^3$.
The diagram 7 on below shows a part of the curve $y = x(x + 2)^3$.



Rajah 7 / Diagram 7

- (a) Cari ungkapan bagi $\frac{dy}{dx}$.
Find an expression for $\frac{dy}{dx}$.
- (b) Cari koordinat titik bagi dua titik pegun P dan Q.
Find the coordinates of the two stationary points, P and Q.
- (c) Seterusnya, tentukan sifat bagi titik pegun P menggunakan kaedah lakaran tangen.
Subsequently, determine the nature of stationary point P by using the tangent sketching method.

[8 markah/ marks]



QUESTION 12

TRIAL
NEGERI TERENGGANU

- (a) Seutas tali panjang 120 cm dibentuk menjadi segi empat tepat dengan panjang x cm dan lebar y cm.

A string with the length of 120 cm is bent to form a rectangle with the length of x cm and width of y cm.

- (i) Tunjukkan bahawa luas segi empat tepat, $A = 60x - x^2$.
Show that the area of rectangle, $A = 60x - x^2$.

- (ii) Seterusnya, cari luas maksimum segi empat tepat itu.
Hence, find the maximum area of the rectangle.

[5 markah]

[5 marks]

- (b) Diberi $y = x + 3x^2$ dan x bertambah dengan kadar $\frac{1}{5}$ unit per saat.

Cari kadar perubahan y apabila $x = 4$.

[2 markah]

Given $y = x + 3x^2$ and x increases at the rate of $\frac{1}{5}$ unit per second.

Find the rate of change of y when $x = 4$.

[2 marks]

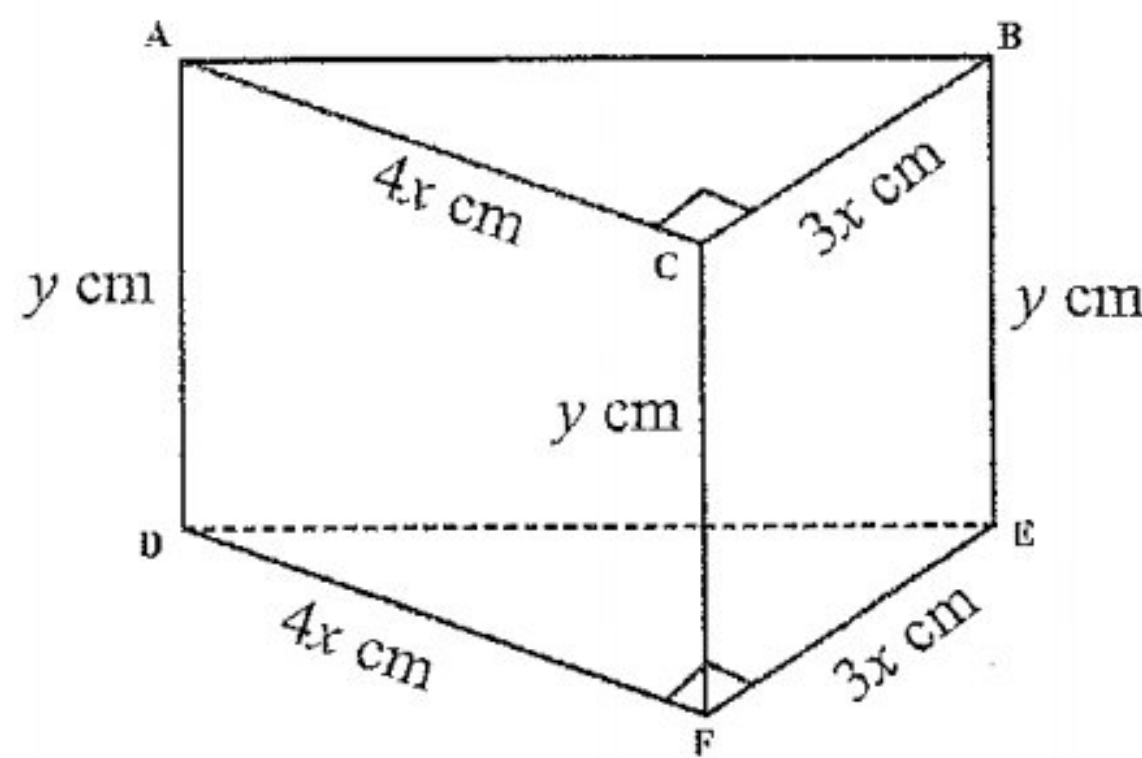


QUESTION 13

TRIAL
YIK-KELANTAN

Rajah 3 menunjukkan sebuah prisma pepejal dengan keratan rentas segi tiga berserenjang dengan sisi selari AD , BE dan CF . Panjang setiap sisi AD , BE dan CF ialah y cm. Segi tiga mempunyai sudut tegak di C dan F . Sisi AC dan DF masing-masing mempunyai panjang $4x$ cm. Sisi BC dan EF masing-masing mempunyai panjang $3x$ cm. Diberi isipadu prisma itu ialah 1500 cm^3 .

Diagram 3 shows a solid prism with triangular cross section perpendicular to the parallel edges AD , BE and CF . The length of each of edges AD , BE and CF is y cm. the triangles have right angles at C and F . The edges AC and DF are each of length $4x$ cm and the edges BC and EF are each of length $3x$ cm. Given the volume of the prism is 1500 cm^3 .



Rajah 3
Diagram 3

- (a) Tunjukkan bahawa jumlah luas permukaan, $S \text{ cm}^2$, diberi oleh
Show that the total surface area, $S \text{ cm}^2$, is given by

$$S = 12x^2 + \frac{3000}{x}$$

[4 markah]

[4 marks]

- (b) Cari nilai pegun bagi S .
The stationary value of S .

[4 markah]

[4 marks]



QUESTION 14

TRIAL
SABK (AGAMA)

Diberi persamaan suatu lengkung ialah $y = x^4 - 4x^3 + 1$.

Given equation of a curve is $y = x^4 - 4x^3 + 1$.

(a) Cari titik-titik pegun bagi garis lengkung itu.

[3 markah]

Find the stationary points of the curve.

[3 marks]

(b) Seterusnya, tentukan sifat setiap titik pegun tersebut dengan menggunakan kaedah lakaran tangen.

[6 markah]

Hence, determine the nature of each stationary points by using the tangent sketching method.

[6 marks]



QUESTION 15

TRIAL
MRSM

Amrin menyertai suatu bengkel membina model geometri. Dia diberi seutas dawai untuk membuat sebuah kerangka kotak berbentuk kuboid. Isipadu kotak itu ialah 375 cm^3 dan panjang tapak kotak adalah dua kali ganda lebar tapaknya.

Amrin participates in a workshop to build a geometric model. He has been given a piece of wire to make a cuboid-shaped box frame. The volume of the box is 375 cm^3 and the base length of the box must be twice its base width.

- (a) Dengan mengambil lebar tapak kotak sebagai x , tunjukkan bahawa panjang dawai yang digunakan adalah $P = 12x + \frac{750}{x^2}$. [3 markah]

By taking the base width of the box as x , show that the length of wire used is $P = 12x + \frac{750}{x^2}$. [3 marks]

- (b) (i) Cari nilai x , dalam cm, yang menjadikan nilai P adalah terpendek.
Find the value of x , in cm, that makes the value of P is the shortest.

- (ii) Seterusnya, kira nilai P tersebut.
Hence, calculate the value of P .

[5 markah]

[5 marks]

- (c) Diberi bahawa harga untuk semeter wayar ialah RM 1.80, berapakah yang harus dibelanjakan oleh penganjur bengkel, dalam RM, untuk 30 orang peserta?

[2 markah]

Given that the price for a meter of a wire is RM 1.80, how much should the organizer of the workshop need to spend, in RM, for 30 participants?

[2 marks]



1

(a) $\frac{dy}{dx} = 8(2x-3)^3$
 $\frac{dy}{dx} = 8(2(1)-3)^3$ K1 ganti nilai x dalam fungsi kecerunan
 $\frac{dy}{dx} = -8$
 $m_1 = -8$
 $-8 \times m_2 = -1$ K1
 $m_2 = \frac{1}{8}$
 $y = \frac{1}{8}x - \frac{25}{8}$ N1

(b) $3 - \frac{3}{x^2} = 0$ K1
 $3x^2 - 3 = 0$
 $x^2 = 1$
 $x = 1$
 $m = 1$ N1

2

(a) (i) tidak tertakrif // tiada jawapan
 undefined // no solution

(ii)

x	-0.1	-0.01	-0.001	-0.0001	0
$f(x)$	-11.9245	-11.9925	-11.9992	-11.9999	-

SS-1 jika nilai dalam jadual tidak tepat kepada 4 tempat perpuluhan

(iii) -12

x	0.1	0.01	0.001	0.0001	0
$f(x)$	-12.0745	-12.0075	-12.0007	-12.0001	-

SS-1 jika nilai dalam jadual tidak tepat kepada 4 tempat perpuluhan

(b) $\frac{dy}{dx} = \frac{2x(x)-(x^2-4)}{x^2}$ ATAU $\frac{dy}{dx} = 1 - 4(-1)(x^{-1-1})$
 $\frac{d^2y}{dx^2} = \frac{2x(x^2)-2x(x^2+4)}{(x^2)^2}$ ATAU $\frac{d^2y}{dx^2} = 4(-2)x^{-2-1}$
 $\frac{dy}{dx} = \frac{x^2+4}{x^2}$ dan $\frac{d^2y}{dx^2} = -\frac{8}{x^3}$
 $x^3 \left(\frac{d^2y}{dx^2} \right) + x^2 \left(\frac{dy}{dx} \right) = x^3 \left(-\frac{8}{x^3} \right) + x^2 \left(\frac{x^2+4}{x^2} \right)$
 $x^3 \left(\frac{d^2y}{dx^2} \right) + x^2 \left(\frac{dy}{dx} \right) = x^2 - 4$
 $(x+2)(x-2) < 0$ dan $-2 < x < 2$

3

(a) (i) tidak tertakrif // tiada jawapan
 undefined // no solution

(ii)

x	-0.1	-0.01	-0.001	-0.0001	0
$f(x)$	-11.9666	-11.9967	-11.9997	-12.0000	-

SS-1 jika nilai dalam jadual tidak tepat kepada 4 tempat perpuluhan

(iii) -12

(b) $\frac{dy}{dx} = (1)(x^2-4) + x(2x)$
 $= 3x^2 - 4$
 $\frac{d^2y}{dx^2} = 6x$
 $x \frac{d^2y}{dx^2} + \frac{dy}{dx} = x(6x) + 3x^2 - 4$
 $x \frac{d^2y}{dx^2} + \frac{dy}{dx} = 9x^2 - 4$
 $9x^2 - 4 > 0$
 $(3x+2)(3x-2) > 0$ dan
 $x < -\frac{2}{3}$ atau $x > \frac{2}{3}$

4

(a) $Z = 2(6-k)^2 + 3k^2$ atau $Z = 2h^2 + 3(6-h)^2$
 Pembezaan, $\frac{dZ}{dk}$ atau $\frac{dZ}{dh}$
 $\frac{dZ}{dk} = 10k - 24$ atau $\frac{dZ}{dh} = 10h - 36$
 Guna $\frac{dZ}{dk} = 0$ atau $\frac{dZ}{dh} = 0$
 $k = \frac{12}{5}$ dan $h = \frac{18}{5}$
 bezakan $\frac{dZ}{dk}$ atau $\frac{dZ}{dh}$ untuk mencari nilai
 $\frac{d^2Z}{dk^2} = 10$ atau $\frac{d^2Z}{dh^2} = 18$
 Titik minimum

(b) $\frac{dy}{dx} = 2(3x-1)(3)$
 $2(3x-1)(3) = -6$
 $(0,1)$

5

(a) $r^2 + (h - 10)^2 = 10^2$ atau $r = \sqrt{10^2 - (h - 10)^2}$
 $A = \pi(10^2 - (h - 10)^2)$ atau $A = \pi(\sqrt{10^2 - (h - 10)^2})^2$
 $A = \pi(20h - h^2)$

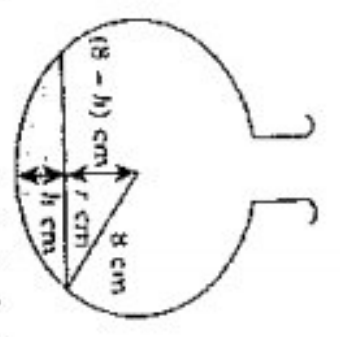
(b) $\frac{dA}{dh} = 20\pi - 2\pi h$
 $20\pi - 2\pi(12)$
 $\frac{dA}{dt} = (20\pi - 2\pi(12)) \times 0.5$
 -2π

(c) $\delta h = \frac{p}{100} \times 12$ atau $p = \frac{\delta h}{12} \times 100$ atau setara
 $\delta A = (20\pi - 2\pi(12)) \times \frac{3}{25} p$
 $-\frac{12p}{25} \pi$

6

(a) $y = x(3 - x)$
 $= 3x - x^2$
 $\frac{dy}{dx} = 3 - 2x$
 $\frac{d^2y}{dx^2} = -2$
 $(3x - x^2)(-2) + 3x - 2x^2 + 12 = 0$
 $-3x + 12 = 0$
 $-3x = -12$
 $x = 4$

(b)(i)



Katakan jejari permukaan membulat air ialah r cm.
 $r^2 = 8^2 - (8 - h)^2$
 $r^2 = 64 - (64 - 16h + h^2)$
 $r^2 = 16h - h^2$
 Luas permukaan membulat air
 $A = \pi r^2$
 $A = \pi(16h - h^2)$
 Tertunjuk

(ii)

$\frac{dA}{dh} = \pi(16 - 2h)$
 Dengan menggunakan petua rantai,
 $\frac{dA}{dt} = \frac{dA}{dh} \times \frac{dh}{dt}$
 $= \pi(16 - 2h) \times -0.5$
 $= \pi[16 - 2(5)] \times -0.5$
 $= -3\pi \text{ cm}^2 \text{ s}^{-1}$

7

(a) $\frac{dy}{dx} = 9x^2 + \frac{1}{x^2}$
 $\frac{dy}{dx} = 10$
 $y - 4 = 10(x + 1)$
 $y = 10x + 14$
 $\pi r^2 h = 24\pi$
 $h = \frac{24}{r^2}$
 $A = \pi r^2 + 2\pi r h + 2\pi r^2$
 $A = 3\pi r^2 + 2\pi r \left(\frac{24}{r^2}\right)$
 $A = 3\pi r^2 + \left(\frac{48\pi}{r}\right)$

(b)(i)

10

(a)	$A = \frac{1}{2}x^2 \left(\frac{100-2x}{x} \right)$	K1
(b)	$A = 50x - x^2$	N1
	(i) $\frac{dA}{dx} = 50 - 2x$	K1
	$50 - 2x = 0$	K1
	$x = 25$	N1
	$\frac{d^2A}{dx^2} = -2$	N1
	(ii) $\theta = \frac{100 - 2(25)}{25}$	K1
	$\theta = 2$	N1

11

(a) $y = x(x+2)^3$
 $\frac{dy}{dx} = x \frac{d}{dx} (x+2)^3 + (x+2)^3 \frac{d}{dx} (x)$
 $= x(3(x+2)^2(1) + (x+2)^3((1)x^{1-1}))$
 $= 2(x+2)^2(2x+1)$

(b) Apabila $\frac{dy}{dx} = 0$,
 $2(x+2)^2(2x+1) = 0$

both $x = -2$
 $y = 0$

$P(-2,0)$
 $Q(-\frac{1}{2}, -\frac{27}{16})$

(c)	x	-2.1	-2	-1.9
	$\frac{dy}{dx}$	-0.064	0	-0.056
	Tanda bagi $\frac{dy}{dx}$	-	0	-
	Lakaran tangen	-	-	-
	Lakaran graf	-	-	-

$P(-2,0)$ ialah titik lengkok balas.

12

(a)(i) $y = 60 - x$ dan $A = x(60 - x)$

$A = 60x - x^2$

(ii) $60 - 2x = 0$

$A = 60(30) - (30)^2$

$A = 900$

(b) $\frac{dy}{dt} = (1 + 6t) \times \left(\frac{1}{5} \right)$

5

(b)

$\frac{d^2y}{dx^2} = 12x^2 - 24x$

Apabila $x = 0$, $\frac{d^2y}{dx^2} = 12(0)^2 - 24(0) = 0$

x	-0.1	0	0.1
$\frac{dy}{dx}$	-0.124	0	-0.116
tanda bagi $\frac{dy}{dx}$	-	0	-
Lakaran tangen	-	-	-
Lakaran graf	-	-	-

Maka, titik (0,1) ialah titik lengkok balas.

Apabila $x = 3$, $\frac{d^2y}{dx^2} = 12(3)^2 - 24(3) = 36 > 0$

Maka, (3,-26) ialah titik minimum.

14

(a)

$y = x^4 - 4x^3 + 1$

$\frac{dy}{dx} = 4x^3 - 12x^2$

$\frac{dy}{dx} = 4x^2(x-3)$

Untuk titik pegun / For stationary points

$\frac{dy}{dx} = 0$

$4x^2(x-3) = 0$

$x = 0$ atau $x = 3$

Apabila / when

$x = 0$, $y = 0^4 - 4(0)^3 + 1 = 1$

$x = 3$, $y = 3^4 - 4(3)^3 + 1 = -26$

Maka, titik pegun ialah / The stationary points are (0,1) dan (3,-26)